

Name KEY

ID KEY

Midterm 3 –60 Points

You must answer all questions. Please write your name on every page. The exam is closed book and closed notes. You may use calculators, but they must not be graphing calculators. No cell phones. Do not use your own scratch paper.

You must show your work to receive full credit

I have neither given nor received unauthorized aid on this examination, nor have I concealed any similar misconduct by others.

Signature KEY

Problem 1

The decision to attend college is a very important one, and family characteristics are considered a major factor in educational attainment. To begin our study of educational attainment, we evaluate the impact of the number of siblings, *sibs*, and the order of birth, *brthord* (1 for first, 2 for second...), on the years of education attained, *educ*.

$$\log(\text{educ}) = \beta_0 + \beta_1 \text{brthord} + \beta_2 \text{sibs} + u$$

Source	SS	df	MS	
Model	.702496954	2	.351248477	Number of obs = 659
Residual	16.2065079	656	.024705042	F(2, 656) = 14.22
				Prob > F = 0.0000
				R-squared = 0.0415
				Adj R-squared = 0.0386
				Root MSE = .15718

logeduc	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
brthord	-.0096447	.005067			
sibs	-.0101424	.0034227			
_cons	2.652929	.0113445			

a.) Please interpret the coefficient on *sibs*. Using the 99% confidence level, please test whether it is significantly different from zero. (10 Points)

Holding birth order constant ^{+B} one additional sibling reduces education attainment by 1.01%.

$H_0: \beta_2 = 0$
 $H_1: \beta_2 \neq 0$ +2

$t_{stat} = \frac{-0.01014 - 0}{0.00342} = -2.96$

$t_{crit} = 2.575$ +1

$|t_{stat}| > t_{crit} \Rightarrow \text{Reject } H_0!$

Sibs has a significant effect on education.

+2

b.) In part (a), I forget about parental education, which is probably quite important. Specifically, I include mother's education, *meduc*, and father's education, *feduc*, as follows:

$$\log(\text{educ}) = \beta_0 + \beta_1 \text{brthord} + \beta_2 \text{sibs} + \beta_3 \log(\text{meduc}) + \beta_4 \log(\text{feduc}) + u$$

The results from estimating this equation are below:

Source	SS	df	MS	Number of obs =	659
Model	3.09146926	4	.772867314	F(4, 654) =	36.58
Residual	13.8175355	654	.02112773	Prob > F =	0.0000
				R-squared =	0.1828
				Adj R-squared =	0.1778
Total	16.9090048	658	.025697576	Root MSE =	.14535

logeduc	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
brthord	-.0009342	.0047804	XX		
sibs	-.0061294	.0031935	XX		
logmeduc	.0752445	.0214354	XX		
logfeduc	.1299058	.017919	XX		
_cons	2.150804	.0520578	XX		

Is this model preferred to the model in 'a'? Please test this at the 95% level, stating your null and alternative hypotheses. Show your work!!! (10 Points)

$SSR_{ur} = 13.82$ $\times 10.5$ $H_0: \beta_3 = 0, \beta_4 = 0$
 $SSR_R = 16.21$ $\times 0.5$ $H_A: H_0 \text{ not true}$ $\times 2$
 $q = 2$ $\times 5$
 $df_{ur} = 654$ $\times 0.5$

$$F_{stat} = \frac{SSR_R - SSR_{ur}}{q} \times \frac{df_{ur}}{SSR_{ur}} = \frac{16.21 - 13.82}{2} \times \frac{654}{13.82} = 56.55$$

$F_{crit} = 3$

$F_{stat} > F_{crit} \Rightarrow$ $\times 2$ Reject H_0 in favor of H_A

c.) Using the regression in 'b', suppose I claim that birth order has a significant effect on education levels. What is the probability that I'm wrong? (10 Points)

$$\begin{aligned}
 P_{\text{value}} &= P_{\sigma} \left(|T| > \left| \frac{-0.0009312}{0.00478} \right| \right) + 11 \\
 &= 2 \left(1 - P_{\sigma}(T < .20) \right) + 11 \\
 &= 2(1 - 0.5793) \approx \boxed{0.845} \times 2
 \end{aligned}$$

d.) Please construct a 90% confidence interval for the coefficient on mother's education. Please interpret this confidence interval. (10 Points)

$$\begin{aligned}
 0.0752 - 1.645 \cdot 0.214 &< B_3 < 0.0752 + 1.645 \cdot 0.214 \\
 + 11 & \qquad \qquad \qquad 0.0399 < B_3 < 0.1104
 \end{aligned}$$

The elasticity of education with respect to mother's education is, with 90% confidence between 0.0399 and 0.1104 + 11

OR with 90% confidence a 1% increase in mother's education yields ~~and~~ between a 0.0399% and 0.1104% increase in education

e.) Please **derive** an equation that allows me to test whether the effect of mother's education is the same as father's education. Along with the derivation, please state the null and alternative hypothesis, and write down any Stata commands required to generate new variables and run the regression. (10 Points)

$$H_0: \beta_3 = \beta_4 \text{ or } \theta = \beta_3 - \beta_4 \Rightarrow \beta_3 = \theta + \beta_4$$

$$H_A: \beta_3 \neq \beta_4 \text{ or } \theta \neq 0$$

$$\log(\text{educ}) = \beta_0 + \beta_1 \text{brthord} + \beta_2 \text{sibs} + \beta_3 \log(\text{meduc}) + \beta_4 \log(\text{feduc}) + u$$

$$= \beta_0 + \beta_1 \text{brthord} + \beta_2 \text{sibs} + (\theta + \beta_4) \log(\text{meduc}) + \beta_4 \log(\text{feduc}) + u$$

$$= \beta_0 + \beta_1 \text{brthord} + \beta_2 \text{sibs} + \theta \log(\text{meduc}) + \beta_4' (\log(\text{meduc}) + \log(\text{feduc})) + u$$

Gen $\log\text{-meduc-feduc} = \log \text{meduc} + \log \text{feduc}$

regress $\log \text{educ}$ brthord sibs $\log \text{meduc}$ $\log\text{-meduc-feduc}$

+ 3

Problem 2

Using the same dataset as before, I estimate the following equation predicting wage outcomes:

$$wage = \beta_0 + \beta_1(educ - 16) + \beta_2(meduc - 12) + \beta_3(feduc - 12) + u$$

where *wage* is the monthly wage of the respondent. The results from estimating this equation are below:

Source	SS	df	MS	
Model	14399607.2	3	4799869.07	Number of obs = 722
Residual	105675187	718	147179.926	F(3, 718) = 32.61
				Prob > F = 0.0000
				R-squared = 0.1199
				Adj R-squared = 0.1162
Total	120074794	721	166539.243	Root MSE = 383.64

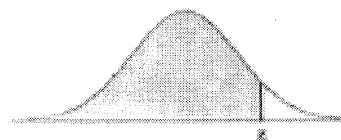
wage	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
educ-16	46.60749	7.168573			
meduc-12	10.67185	6.272866			
feduc-12	11.04096	5.510691			
_cons	1118.009	20.69534			

Please construct a 92% confidence interval for the constant. Please interpret this confidence interval. Show your work!!! (10 Points)

$$1118.009 - 20.695 \cdot 1.75 < \beta_0 < 1118.009 + 20.695 \cdot 1.75$$

$$1081.793 < \beta_0 < 1154.225 \quad \#4$$

With 92% confidence, a person with 16 years of education, a mother with 12 years of education, and a father with 12 years of education will earn between \$1081.79 and \$1154.22 #6



Normal Distribution from $-\infty$ to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

TABLE G.3b

5% Critical Values of the F Distribution

		Numerator Degrees of Freedom									
	1	2	3	4	5	6	7	8	9	10	
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	

Example: The 5% critical value for numerator $df = 4$ and large denominator $df(\infty)$ is 2.37.

Source: This table was generated using the Stata® function invFtail.